The calculation of uncertainty of the calibration results for a torque measuring device calibrated to British Standard BS 7882: 2008.

Barry C. Pratt
Head of Laboratory
Norbar Torque Tools Ltd
Beaumont Road
Banbury
Oxfordshire
OX16 1XJ
United Kingdom

Phone: + 44 (0) 1295 270 333
E-mail: bpratt@norbar.com

Abstract – BS 7882:2008[1] annex B gives a "Method example of determining uncertainty of the calibration results of the toque measuring device".

The calculation of uncertainties associated with the calibration results is explained and includes assessment of the likely causes of uncertainty; their potential magnitude; and the type of distribution associated with each element. The combination of each uncertainty into the final expanded uncertainty is then demonstrated by the use of a worked example.

Introduction

It is a requirement for all accredited Calibration Laboratories that results reported in a Calibration Certificate are accompanied by a statement describing the uncertainty associated with those results. Without such an indication results cannot be compared either amongst themselves or with reference values.

The expanded uncertainties are derived from the calibration results and associated type B (systematic) uncertainty contributions in accordance with the requirements of BS 7882:2008 annex B.

In addition to the uncertainty contributions referenced in BS 7882:2008 sources of uncertainty specific to unsupported calibration beams are included. Depending on individual circumstances uncertainty contribution may come from other sources, table 1 gives typical examples. It is the responsibility of the laboratory concerned to identify the sources of uncertainty that are present during a calibration and to formulate their uncertainty budgets accordingly.

A worked example for a 1000 N·m. transducer calibrated in accordance with BS 7882:2008 Class 0.2 increasing torques only is given. All sources of uncertainty are considered uncorrelated input quantities.

Table1. Sources of uncertainty and assumed distributions.

SOURCE OF UNCERTAINTY	DISTRIBUTION	DEVISOR
Uncertainty due to the application of torque	Normal	2
Uncertainty due to repeatability.	Rectangular	√3
Uncertainty due to reproducibility.	U shaped	√2
Uncertainty due to resolution.	Rectangular	√3
Uncertainty due to residual deflection.	Rectangular	√3
Uncertainty due to temperature variations	Triangular	√6
Uncertainty due to the error of interpolation.	Triangular	√6
Uncertainty due to reversibility.	Rectangular	√3
Uncertainty due to the transducers susceptibility to bending effects	Rectangular	√3
Uncertainty due to the axis of the applied torque not being horizontal.	Rectangular	√3
Uncertainty due to voltage ratio measurement	Normal	2
Uncertainty due to voltage ratio meter uncorrected errors.	Rectangular	√3
Uncertainty due to eccentric loading	Rectangular	√3
Uncertainty due to friction effects	Rectangular	√3

General requirements

The calibration uncertainty is that associated with the mean indicated deflection of a calibration for a given applied torque. It may be given in torque units (N·m), electrical signal output (mV/V) (V) or as a relative value in percentage. For the purpose of this paper relative values are used as the parameters for the device are expressed in percentage.

At each calibration torque *T*, a combined standard uncertainty u_c is calculated from the readings obtained during the calibration and type b uncertainty contributions. These uncertainties are then multiplied by the coverage factor $k = 2$ to give an expanded uncertainty value *U* providing a level of confidence of approximately 95%.

$$u_c = \sqrt{\sum_{i=1}^{n} u_i^2}$$ n = number of uncertainty contributions u_i.

$$U = k \cdot u_c$$

and where :

u_1 = standard uncertainty associated with the calibration torque;

u_2 = standard uncertainty associated with the reproducibility of the device;

u_3 = standard uncertainty associated with the repeatability of the device;

u_4 = standard uncertainty associated with the resolution of indicator;

u_5 = standard uncertainty associated with the residual deflection of the device;

u_6 = standard uncertainty associated with the temperature of the device;

u_7 = standard uncertainty associated with the horizontal axis of an unsupported calibration beam.

u_8 = standard uncertainty associated with the devices susceptibility to bending effects.

Calibration torque uncertainty, u_1

u_1 is the expanded uncertainty associated with the torques generated by the calibration beam (supported or unsupported) or reference device, and is subject to its own uncertainty budget. This value is normally obtained from the calibration certificate of the calibration beam or from the manufacturer. For the Norbar unsupported calibration beam used in this example it is assumed to be ± 0.02% using a coverage factor of k = 2 to give a level of confidence of approximately 95%.

Calculation of reproducibility uncertainty, u_2

u_2 is the uncertainty contribution due to the reproducibility of the measured deflection. It can be assumed that, at each calibration torque T:

$$u_2 = \frac{0.5R_2}{\sqrt{2}}$$

where R_2 is the relative reproducibility defined in 5.3 of BS7882: 2008

Calculation of repeatability uncertainty, u_3

u_3 is the uncertainty contribution due to the repeatability of the measured deflection. It can be assumed that, at each calibration torque T:

$$u_3 = \frac{0.5R_1}{\sqrt{3}}$$

where R_1 is the relative repeatability defined in 5.2 of BS7882: 2008

Calculation of resolution uncertainty, u_4

Each deflection value is calculated from two readings (the reading with an applied torque minus the reading at zero torque). Because of this, the resolution of the indicator needs to be included twice as a single triangular distribution with a standard uncertainty of $r/\sqrt{6}$, where r is the resolution expressed as a relative value. Where it is possible to tare the initial reading of each series to zero, a single rectangular distribution with a standard uncertainty of $r/\sqrt{12}$ can be used.

$$u_4 = \frac{r}{\sqrt{6}} \qquad \text{or} \qquad u_4 = \frac{r}{\sqrt{12}}$$

Calculation of residual deflection uncertainty, u_5

u_5 is the uncertainty contribution due to the variation in the relative residual deflection R_0.

$$u_5 = \frac{0.5R_0}{\sqrt{3}}$$

where R_0 is the relative residual deflection defined in 5.5 of BS7882: 2008

Calculation of temperature uncertainty, u_6

u_6 is the uncertainty contribution due to the variation of temperature throughout the calibration, together with the uncertainty in the measurement of the calibration temperature. The sensitivity of the device to temperature needs to be determined:

$$u_6 = \frac{K\Delta t}{2\sqrt{3}} \qquad \text{or} \qquad u_6 = \frac{K\Delta t}{\sqrt{6}}$$

where:

K is the device's relative temperature coefficient expressed as a percentage of maximum applied torque per degree Celsius, derived either by tests or from the manufacturer's specifications; NOTE The temperature coefficient to be used is that of sensitivity, not of zero.
Δt is the calibration temperature range, allowing for the uncertainty in the measurement of the temperature.

How to determine the sensitivity of the device to temperature is given in the NPL good practice guide No.107 "Guide to the calibration and testing of torque transducers"[2].

Calculation of uncertainty due to the axis of an unsupported calibration beam not being horizontal, u_7

u_7 is the uncertainty contribution due to the axis of the calibration beam not being horizontal H_{axis}. The horizontal axis of the applied torque is allowed to vary ± 1°. This error causes a loss of applied torque (u_1) of 0.015%. NOTE for ± 2° this value would be 0.061%.

$$u_7 = \frac{H_{axis}}{\sqrt{3}}$$

Calculation of uncertainty due to the devices susceptibility to bending effects, u_8

u_8 is the uncertainty contribution due to bending effects D_b. The torque measuring device may show susceptibility to bending effects Determined from tests as 0.1%.

$$u_8 = \frac{D_b}{\sqrt{3}}$$

How to determine the susceptibility to bending of the device is given in the NPL good practice guide No.107 “Guide to the calibration and testing of torque transducers“[2][3].

Calculation of combined standard uncertainty, u_c

For each calibration torque the combined standard uncertainty u_c is calculated by combining the individual standard uncertainties in quadrature.

$$u_c = \sqrt{\sum_{i=1}^{n} u_i^2}$$

Calculation of expanded uncertainty, U

$$U = k \cdot u_c$$

where $k = 2$.

Overall accuracy of the device, O_a

The overall accuracy O_a of the device expressed as an uncertainty can be obtained by combining the classification accuracy with the expanded uncertainty in quadrature.

$$O_a = \sqrt{\left(\frac{E_i}{\sqrt{3}}\right)^2 + \left(\frac{U}{2}\right)^2}$$

The overall accuracy of the device is then multiplied by the coverage factor $k = 2$ to give an expanded uncertainty value UO_a.

UO_a encompasses all reported errors which subsequently do not need to be corrected for.

Classification accuracy is obtained from the error of indication E_i in Table 2 of BS7882: 2008.

An alternative approach is to directly add the actual reported value of E_i to U. In this instance the sign of E_i would be ignored and assumed to be a ± value.

$UO_a = E_i + U$

Worked example of calculation of uncertainty of the calibration results of a torque measuring device calibrated in torque units (increasing torques only)

Classification to BS 7882: 2008

Class 0·2 from 1000·0 N·m to 100·0 N·m

Applied Torque N.m	Raw data 0°		Raw data 90°	Mean indicated deflection
	1	2	3	
0	0.0	0.0	0.0	
100	100.0	99.9	99.9	**100.0**
200	199.9	199.9	199.8	**199.9**
400	399.8	399.7	399.7	**399.8**
600	599.8	599.7	599.7	**599.8**
800	799.9	799.8	799.7	**799.8**
1000	1000.0	999.9	999.8	**999.9**
0	0.00	0.01	0.01	

Applied Torque N.m	Relative Repeatability R_1	Relative Reproducability R_2	Relative Error of Indication E_i	Relative residual deflection R_0
0				
100	0.100%	0.100%	-0.050%	
200	0.000%	0.050%	-0.075%	
400	0.025%	0.025%	-0.063%	
600	0.017%	0.017%	-0.042%	
800	0.013%	0.025%	-0.025%	
1000	0.010%	0.020%	-0.010%	
0				0.001%

Applied Torque N.m	Classification: Relative Repeatability R_1	Relative Reproducibility R_2	Error of indication E_i	Relative residual deflection R_0	Lower limit of calibration
100	0.2	0.10	0.100		0.2
200	0.05	0.05	0.200		0.1
400	0.05	0.05	0.200		0.05
600	0.05	0.05	0.100		0.05
800	0.05	0.05	0.050		0.05
1000	0.05	0.05	0.050	0.05	0.05

Uncertainties calculated in accordance with BS7882: 2008 annex B

	Applied torque		Reproducibility		Repeatability		Resolution		Residual deflection	
Uncertainty contribution	u_1		u_2		u_3		u_4		u_5	
Formula	Taken from the calibration certificate of the calibration machine.		$u_2 = \frac{R_2 \times 0.5}{\sqrt{2}}$		$u_3 = \frac{R_1 \times 0.5}{\sqrt{3}}$		$u_4 = \frac{r}{\sqrt{12}}$		$u_5 = \frac{0.5 \times R_0}{\sqrt{3}}$	
0	Value	Uncertainty	Value	Uncertainty	Value	Uncertainty	Value	Uncertainty	Value	Uncertainty
100	0.020%	0.010%	0.100%	0.035%	0.100%	0.029%	0.100%	0.029%		
200	0.020%	0.010%	0.050%	0.018%	0.000%	0.000%	0.050%	0.014%		
400	0.020%	0.010%	0.025%	0.009%	0.025%	0.007%	0.025%	0.007%		
600	0.020%	0.010%	0.017%	0.006%	0.017%	0.005%	0.017%	0.005%		
800	0.020%	0.010%	0.025%	0.009%	0.013%	0.004%	0.013%	0.004%		
1000	0.020%	0.010%	0.020%	0.007%	0.010%	0.003%	0.010%	0.003%	0.001%	0.0003%

	Temperature		Horezontal axis error		Bending effects			
Uncertainty contribution	u_6		u_7		u_8		Combined uncertainty	Expanded Uncertainty K = 2
Formula	$u_6 = \frac{K \times \Delta t}{2 \times \sqrt{3}}$		$u_7 = \frac{H_{axis}}{\sqrt{3}}$		$u_8 = \frac{D_b}{\sqrt{3}}$		$u_c = \sqrt{\sum_{i=1}^{8} u_i^2}$	$U = u_c \times 2$
0	Value	Uncertainty	Value	Uncertainty	Value	Uncertainty		
100	0.035%/°C	0.020%	0.015%	0.009%	0.10%	0.058%	0.083%	0.165%
200	0.035%/°C	0.020%	0.015%	0.009%	0.10%	0.058%	0.067%	0.133%
400	0.035%/°C	0.020%	0.015%	0.009%	0.10%	0.058%	0.064%	0.128%
600	0.035%/°C	0.020%	0.015%	0.009%	0.10%	0.058%	0.063%	0.126%
800	0.035%/°C	0.020%	0.015%	0.009%	0.10%	0.058%	0.063%	0.127%
1000	0.035%/°C	0.020%	0.015%	0.009%	0.10%	0.058%	0.063%	0.126%

Example uncertainty budgets for a 1000 N·m. transducer calibrated in accordance with BS 7882:2008 at 60% of full scale are given. NOTE a triangular distribution is assumed for temperature and a rectangular distribution for resolution base on ± ½ a least significant digit.

EXAMPLE ONLY

CALIBRATION LABORATORY WORKSHEET FOR UNCERTAINTY CALCULATIONS

UNCERTAINTY BUDGET FOR 1000 N·m TRANSDUCER @ 60% OF FULL SCALE SYSTEM CALIBRATION IN N·m

BCP8B

Symbol	Source of uncertainty	value ±	Probability distribution	Divisor	c*i*	u*i* (TRAN) ± % (units)	v*i* or v*eff*
T_Q	TORQUE APPLICATION	0.02	Normal	2	1	.01	
$T_{mV/V.sys}$	VARIATION DUE TO TEMPERATURE CHANGE	2°C	Triangular	√6	0.035% per °C	.029	
H_{axis}	ERROR DUE TO THE AXIS OF APPLIED TORQUE NOT BEING HORIZONTAL	0.015%	Rectangular	√3	1	.009	
D_{RE}	ERROR DUE TO RESOLUTION	.0083%	Rectangular	√3	1	.005	
R_0	ERROR DUE TO RESIDUAL DEFLECTION	0.001%	Rectangular	√3	1	0.0003	
R_1	ERROR DUE TO REPEATABILITY	0.0085%	Rectangular	√3	1	0.005	
R_2	ERROR DUE TO REPRODUCIBILITY	0.0085%	U	√2	1	0.006	
R_3	ERROR DUE TO REVERSIBILITY	0.025%	Rectangular	√3	1	.014	
D_b	ERROR DUE TO BENDING	0.10%	Rectangular	√3	1	.058	
u*c* (TRAN)	Root, Sum, Squ. U*i* (TRAN) Combined uncertainty		normal			0.068	
U	Expanded uncertainty		normal (k=2)			0.14	

NAME	B. C. PRATT	DATE	12.8.2008

EXAMPLE ONLY

CALIBRATION LABORATORY WORKSHEET FOR UNCERTAINTY CALCULATIONS

UNCERTAINTY BUDGET FOR 1000 N·m TRANSDUCER @ 60% OF FULL SCALE CALIBRATION IN mV/V

BCP8B

Symbol	Source of uncertainty	value ±	Probability distribution	Divisor	c*i*	u*i* (TRAN) ± % (units)	v*i* or v*eff*
T_Q	TORQUE APPLICATION	0.02	Normal	2	1	.01	
$T_{mV/V.sys}$	VARIATION DUE TO TEMPERATURE CHANGE	2°C	Triangular	√6	0.035% per °C	.029	
H_{axis}	ERROR DUE TO THE AXIS OF APPLIED TORQUE NOT BEING HORIZONTAL	0.015%	Rectangular	√3	1	.009	
UN_{Vratio}	UNCERTAINTY OF VOLTAGE RATIO MEASUREMENT	0.01%	Normal	2	1	.005	
M_{mer}	VOLTAGE RATIO METER UNCORRECTED ERRORS	0.006%	Rectangular	√3	1	.0035	
D_{RE}	ERROR DUE TO RESOLUTION	.008%	Rectangular	√3	1	.005	
R_0	ERROR DUE TO RESIDUAL DEFLECTION	0.001%	Rectangular	√3	1	0.0003	
R_1	ERROR DUE TO REPEATABILITY	0.0085%	Rectangular	√3	1	0.005	
R_2	ERROR DUE TO REPRODUCIBILITY	0.0085%	U	√2	1	0.006	
E_{it}	ERROR OF INTERPOLATION	0.00019%	Triangular	√6	1	.00008	
D_b	ERROR DUE TO BENDING	0.10%	Rectangular	√3	1	.058	
u*c* (TRAN)	Root, Sum, Squ. U*i* (TRAN) Combined uncertainty		normal			0.066	
U	Expanded uncertainty		normal (k=2)			0.13	

NAME	B. C. PRATT	DATE	12.8.2008

References

[1] BS7882:2008; Calibration and classification of torque measuring devices

[2] Robinson A, 2008 Guide to the calibration and testing of torque transducers. NPL. National measurement good practice guide No.107.

[3] Pratt B, Robinson A, 2006 A comparison between supported and unsupported beams for use in static torque calibrations. *Proceedings of the* 18th Imeko World Congress, Rio de Janeiro.

[4]. ISO TAG 4 Guide to the Expression of Uncertainty in Measurement, BIPM, IEC, IFCC, ISO, IUPAC, OIML. International Organisation for Standardization, Geneva, Switzerland, First Edition, 1993.

[5] The Expression of Uncertainty and Confidence in Measurement. United Kingdom Accreditation Service, 21 – 47 High Street, Feltham, Middlesex, TW13 4UN, UK. M3003, Edition 2, 2007.

[6] Pratt B.C., Sources of uncertainty of torque generation using masses hanging from and unsupported calibration beam. Issue 4, 24 July 2001.